

0D Single-Zone Model of a 4-Stroke SI Engine¹

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May 2025

¹Source code and MATLAB App available at https://github.com/batacoglu/MATLAB_OD_engine_simulation_app

Abstract

A single-zone, zero-dimensional (0D) model of a spark ignition (SI) internal combustion engine (ICE) was modeled in MATLAB. The simulation computes instantaneous cylinder volume, piston kinematics, trapped mass, heat-release via a Wiebe sub-model, in-cylinder temperature and pressure histories, indicated and brake performance metrics, including friction losses. A user interface was implemented for changing default parameters and visualization of output graphs.

1 Introduction

Spark-ignition (SI) engine models are essential tools for understanding thermodynamic performance and emissions. A zero-dimensional (0D) single-zone model balances simplicity and physical insight by treating the combustion chamber as a uniform body of gas with time-varying volume and heat input. This project implements such a model in MATLAB, using established correlations for heat transfer (Woschni), a Wiebe function for heat release, and empirical friction models to predict brake performance. This report details the mathematical formulation, simulation methodology, and resulting thermodynamic characteristics of the engine.

2 Methodology

To simulate the performance of the spark ignition (SI) engine, a zero-dimensional (0D) single-zone thermodynamic model was developed. This approach assumes that the cylinder content is a homogeneous mixture of air and fuel, with spatial gradients in temperature and pressure neglected.

2.1 Governing Equations

The thermodynamic state of the cylinder at any crank angle θ is governed by the First Law of Thermodynamics for an open system:

$$\frac{dU}{d\theta} = \frac{dQ_{comb}}{d\theta} - \frac{dQ_{wall}}{d\theta} - P \frac{dV}{d\theta} + \sum \dot{m}_i h_i \quad (1)$$

Where U represents internal energy, Q_{comb} is the heat released by combustion, and Q_{wall} accounts for heat transfer to the cylinder walls. The final term, $\sum \dot{m}_i h_i$, accounts for enthalpy carried by mass crossing the system boundary; since the simulation only integrates the closed-valve interval between IVC and EVO, no mass crosses the boundary during this period and this term is identically zero throughout the simulated cycle. The cylinder volume $V(\theta)$ is defined by the slider-crank mechanism geometry:

$$V(\theta) = V_c + \frac{\pi B^2}{4} s(\theta) \quad (2)$$

Where V_c is the clearance volume, B is the bore, L is the connecting rod length, r is the crank radius, and $s(\theta)$ is the instantaneous piston position, measured from the TDC.

$$s(\theta) = r(1 - \cos \theta) + L - \sqrt{L^2 - (r \sin \theta)^2} \quad (3)$$

To accurately model the convective heat losses to the cylinder walls and the mechanical friction of the engine, the instantaneous piston velocity $v_p(\theta)$ is required. Applying the chain rule with the engine angular velocity $\omega = d\theta/dt$, the instantaneous piston velocity is derived by differentiating the piston displacement profile with respect to the crank angle:

$$v_p(\theta) = \omega \frac{ds}{d\theta} = \omega \left[r \sin \theta + \frac{r^2 \sin \theta \cos \theta}{\sqrt{L^2 - (r \sin \theta)^2}} \right] \quad (4)$$

Where ω is the angular velocity expressed in radians per second. In addition to the instantaneous velocity, the mean piston speed $\bar{S}_p$ is a key parameter for characterizing how frictional losses and convective heat transfer scale with engine speed, and it provides a useful reference velocity when interpreting the Woschni correlation, which uses the instantaneous local gas velocity U_p directly. It is evaluated using the engine stroke S and the rotational speed N in revolutions per minute (RPM):

$$\bar{S}_p = \frac{2SN}{60} \quad (5)$$

2.2 Combustion and Heat Transfer Models

To close the system of equations, two empirical sub-models were integrated.

Combustion: The fuel energy release rate is dictated by the Wiebe function, which characterizes the mass fraction burned (x_b) as a function of crank angle:

$$x_b(\theta) = 1 - \exp \left[-a \left(\frac{\theta - \theta_{start}}{\Delta\theta} \right)^{m+1} \right] \quad (6)$$

Heat Transfer: Wall heat losses are calculated using the Woschni correlation, which relates the convective heat transfer coefficient to the instantaneous cylinder pressure, temperature, and mean piston speed.

$$h = C_w \cdot B^{-0.2} \cdot \left(\frac{P}{P_{ref}} \right)^{0.8} \cdot \left(\frac{T}{T_{ref}} \right)^{-0.53} \cdot U_p^{0.8} \quad (7)$$

Where C_w is a global calibration coefficient, P_{ref} and T_{ref} are reference pressure and temperature used to nondimensionalize the correlation, and U_p is a characteristic gas velocity derived from the piston motion.

2.3 Performance Metrics and Friction

Once the cylinder pressure $P(\theta)$ and volume $V(\theta)$ are solved over the closed-valve interval of the engine cycle, the indicated performance metrics are derived by integrating the pressure over the volume. The Indicated Mean Effective Pressure (IMEP) is calculated as:

$$\text{IMEP} = \frac{1}{V_d} \int_{IVC}^{EVO} P dV \quad (8)$$

Where V_d is the displacement volume. To account for the transition from the cylinder to the output shaft, mechanical friction losses are modeled using an empirical correlation, expressed as a Friction Mean Effective Pressure (FMEP) that scales linearly with engine speed N :

$$\text{FMEP} = A + B \cdot N \quad (9)$$

Where $A = 1.5 \text{ bar}$ is a speed-independent baseline term and $B = 1 \times 10^{-4} \text{ bar/rpm}$ scales friction with rotational speed. No quadratic speed term was included in this baseline calibration.

The final brake performance is obtained by subtracting these losses from the indicated power (P_{ind}):

$$P_{brake} = P_{ind} - P_{fric} \quad (10)$$

This model creates a closed-loop simulation that allows for the iterative adjustment of engine parameters such as ignition timing and compression ratio to observe their immediate effects on efficiency and power output.

3 Simulation Setup

Toyota 1NZ-FE, 1.497 L inline-4 gasoline engine was chosen for this simulation. Its specs are as follows:

Table 1: Toyota 1NZ-FE Engine Parameters

Parameter	Value
Bore (B)	75.0 mm
Stroke (S)	84.7 mm
Connecting rod length (L)	140.85 mm
Compression ratio (CR)	10.5
Displacement per cylinder (V_d)	374.1 cm ³
Clearance volume (V_c)	39.4 cm ³

3.1 Instantaneous Volume

Plugging parameters found in Table 1 into Equation (2) and Equation (3) yields Figure 1.

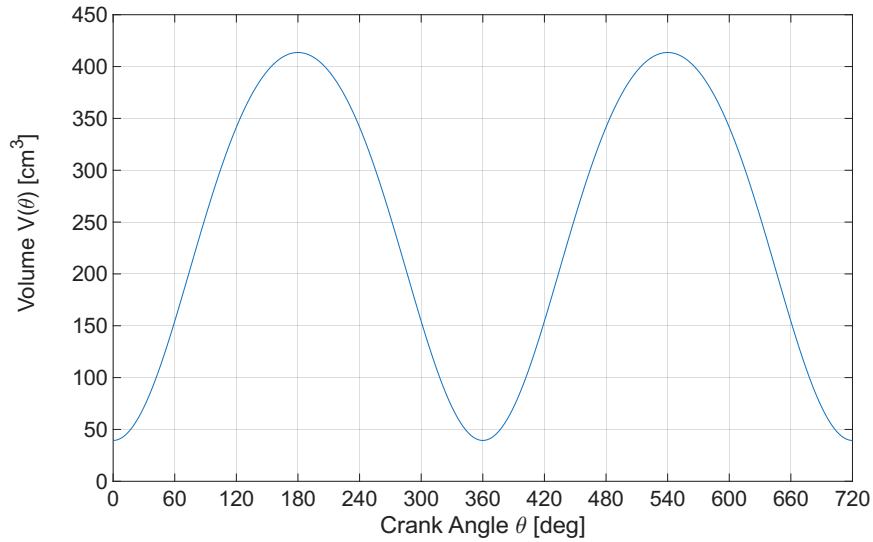


Figure 1: Instantaneous cylinder volume (0° to 720°)

3.2 Piston Kinematics

Similar to the volume calculation, the instantaneous piston velocity is determined by applying the engine speed to the geometric parameters. Operating at an engine speed of 3000 RPM, the velocity profile is calculated using Equation (4) and is presented in Figure 2.

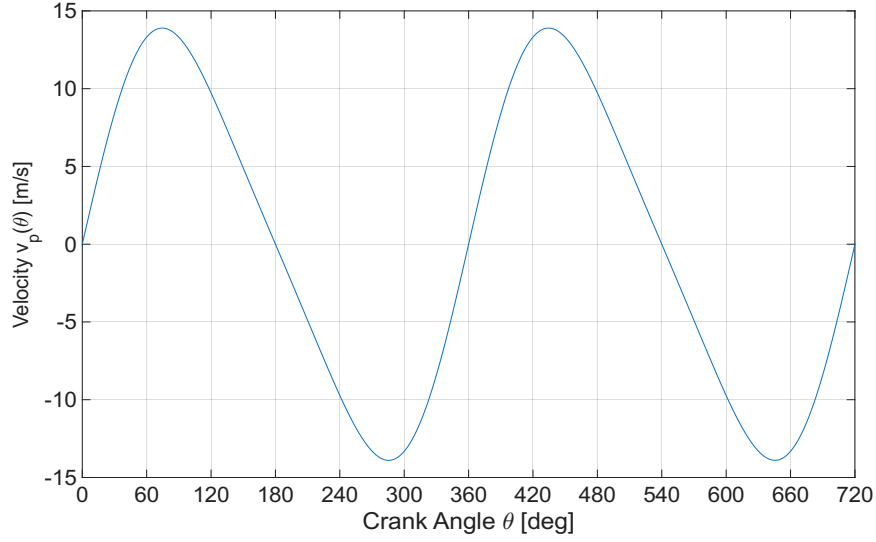


Figure 2: Instantaneous piston velocity at 3000 RPM (0° to 720°)

From the kinematic profile, the mean and peak piston speeds can be extracted. At the simulated operating speed of 3000 RPM, the mean piston speed ($\bar{S}_p$) using Equation (5) evaluates to 8.47 ms^{-1} . The peak instantaneous piston velocity reaches a maximum of 13.90 ms^{-1} during the downward strokes. These velocity magnitudes are within standard limits for passenger vehicle engines and serve as direct inputs for the friction mean effective pressure (FMEP) and Woschni heat transfer calculations.

3.3 Initial Conditions and Trapped Mass

To initiate the closed-cycle thermodynamic simulation (comprising the compression, combustion, and expansion strokes), the state of the gas inside the cylinder at Intake Valve Closing (IVC) must be explicitly defined. The simulation assumes that the pressure and temperature at IVC approximate the intake manifold conditions. For this simulation, Intake Valve Closing was set to 20° after Bottom Dead Center (200° crank angle), and Exhaust Valve Opening was set to 40° before Bottom Dead Center on the expansion side (500° crank angle), defining the closed-valve interval over which the energy balance is integrated.

Using the Ideal Gas Law, the total trapped mass m_{cyl} per cylinder is calculated as:

$$m_{cyl} = \frac{P_0 \cdot V_{IVC}}{R \cdot T_0} \approx 0.47 \text{ g} \quad (11)$$

Where R is the specific gas constant for air ($287 \text{ J kg}^{-1} \text{ K}^{-1}$). For this naturally aspirated baseline model, the intake pressure P_0 is set to ambient conditions (1.0 bar)

and the initial temperature T_0 is set to 300 K. Once m_{cyl} is determined, the mass remains constant throughout the closed-valve portion of the cycle, allowing the explicit Euler integration scheme to solve for the pressure and temperature at each subsequent crank angle step.

3.4 Combustion and Heat Release

With the trapped mass established, the total thermal energy available per cycle (Q_{in}) is calculated based on a stoichiometric air-fuel ratio (AFR = 14.7) and a standard gasoline Lower Heating Value (LHV) of 44 MJ kg^{-1} .

To model the transient combustion process, the empirical Wiebe function was employed. The mass fraction burned (x_b) and the corresponding rate of heat release are parameterized by the ignition timing (θ_{start}), combustion duration ($\Delta\theta$), and two dimensionless shape factors (a and m). For this simulation, the ignition timing was advanced to 350° crank angle (10° Before Top Dead Center) with a burn duration of 40° . Standard SI engine shape factors of $a = 5$ and $m = 2$ were selected to represent a typical turbulent flame propagation profile.

Applying these parameters to the Wiebe formulation yields the mass fraction burned and the derivative heat-release rate ($dQ/d\theta$), illustrated in Figure 3 and Figure 4, respectively.

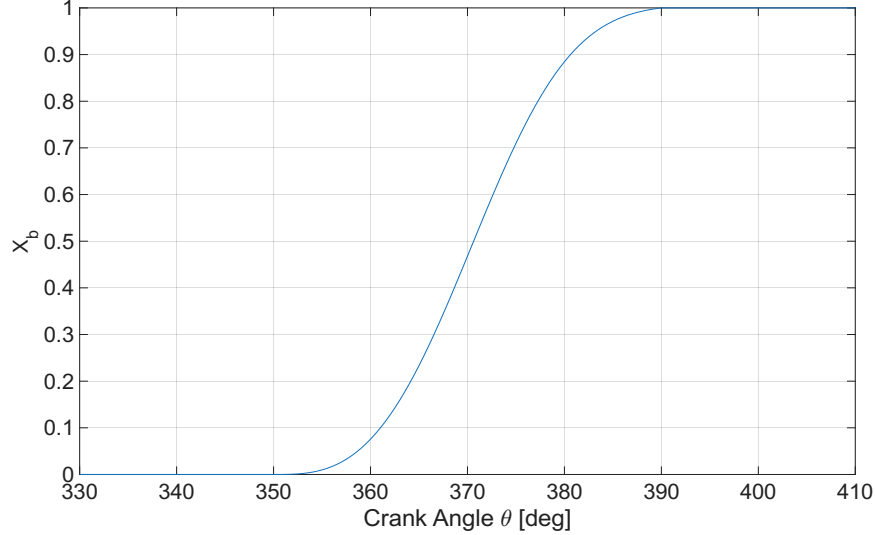


Figure 3: Wiebe mass fraction burned profile ($a = 5, m = 2$)

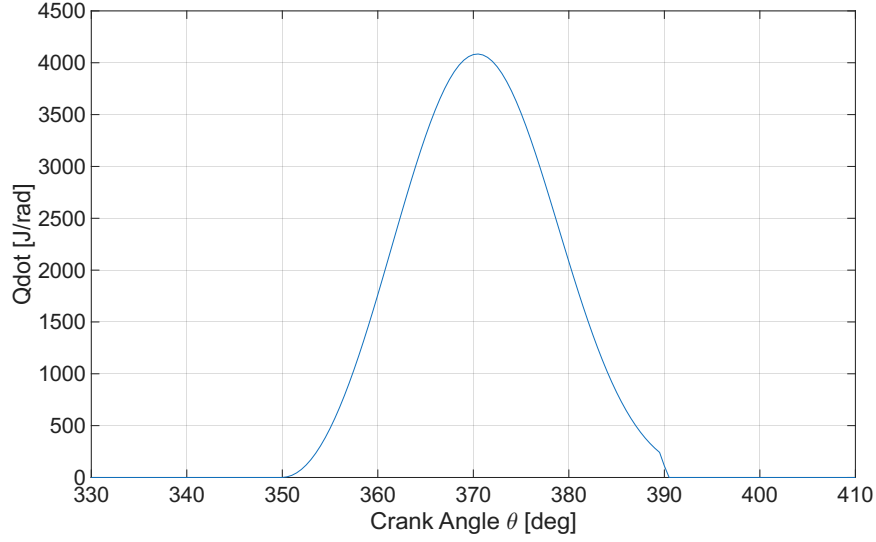


Figure 4: Instantaneous heat-release rate during combustion (350° to 390°)

3.5 Heat Transfer and Numerical Integration

The final component of the simulation framework couples the heat release with the convective heat losses to the cylinder walls. Rather than utilizing piecewise coefficients, a unified Woschni heat transfer correlation was implemented to compute the convective heat transfer coefficient h dynamically at each crank angle:

$$h = C_w \cdot B^{-0.2} \cdot \left(\frac{P}{P_{ref}} \right)^{0.8} \cdot \left(\frac{T}{T_{ref}} \right)^{-0.53} \cdot U_p^{0.8} \quad (12)$$

Where P_{ref} is the reference pressure (1×10^5 Pa), T_{ref} is the reference temperature (300 K), and U_p is the modified local gas velocity defined as $U_p = |v_p| + 2$. Because the Woschni correlation is highly empirical, the global heat transfer multiplier C_w serves as a critical tuning parameter. For this simulation, C_w was set to 130 to calibrate the total heat rejection.

The convective heat loss rate is then evaluated using Newton's law of cooling,

$$\dot{Q}_{wall} = h \cdot A_{tot} \cdot (T - T_{wall}) \quad (13)$$

where T_{wall} is the assumed constant cylinder wall temperature (400 K), representing the cooled metal surface in contact with the in-cylinder gas. The heat transfer area A_{tot} is approximated as the sum of the piston crown area and the instantaneous liner area exposed above the piston, $A_{tot} = A_p + \pi B(H_c + s(\theta))$, where $H_c = V_c/A_p$ is an effective clearance height; the cylinder head area is not modeled separately in this simplified formulation.

With all geometric and empirical sub-models defined, the First Law of Thermodynamics Equation (1) is evaluated as a system of ordinary differential equations (ODEs). Because the state variables change rapidly during combustion, an explicit Euler integration scheme with a discrete step size of $\Delta\theta = 0.5^\circ$ is utilized. The simulation marches forward from the established IVC boundary conditions at 200° through to EVO at 500° :

$$T_{i+1} = T_i + \left(\frac{dT}{d\theta} \right)_i \Delta\theta \quad (14)$$

$$P_{i+1} = P_i + \left(\frac{dP}{d\theta} \right)_i \Delta\theta \quad (15)$$

This numerical loop yields the complete in-cylinder pressure and temperature histories, which serve as the foundation for the indicated and brake performance analysis.

3.6 Interactive Application

To facilitate rapid parameter exploration, the simulation was additionally packaged as a MATLAB App Designer application. The interface exposes all key model inputs, including engine geometry (bore, stroke, connecting rod length, compression ratio), operating conditions (engine speed, intake pressure and temperature), combustion parameters (ignition timing, burn duration), IVC/EVO timing, and heat transfer calibration constants (cylinder wall temperature). Upon setting the desired parameters and initiating the simulation, the application generates the complete set of instantaneous and cycle-averaged results described in Sections 3 and 4, including cylinder volume, piston kinematics, Wiebe combustion profiles, in-cylinder pressure and temperature histories, the normalized P-V diagram, and the brake power sweep, each rendered in a dedicated tab for interactive inspection.

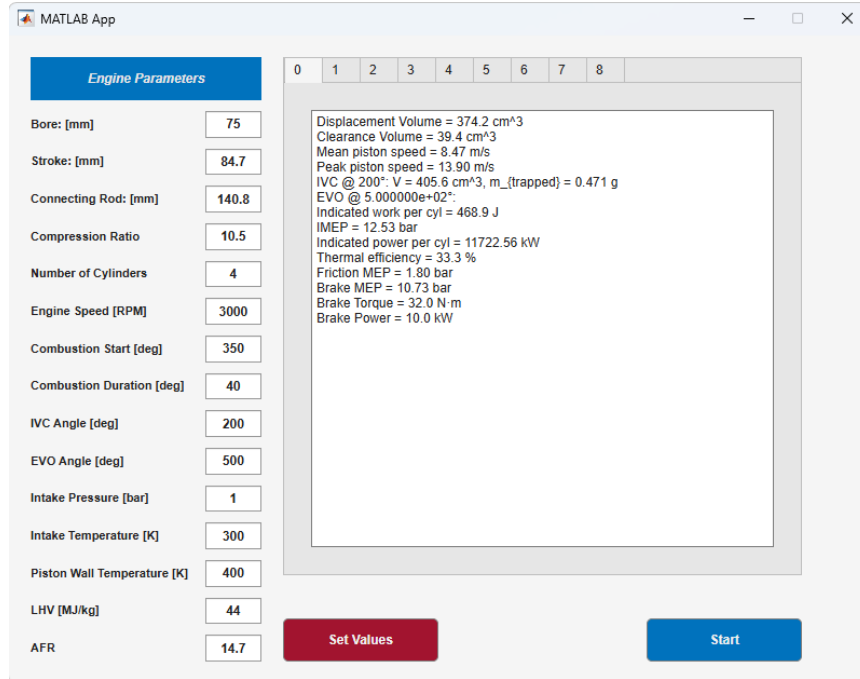


Figure 5: UI of the interactive application. Can be found inside GitHub.

4 Results and Discussion

With the boundary conditions set and the First Law differential equations integrated over the closed-valve interval from IVC (200°) to EVO (500°), the core thermodynamic properties of the 1NZ-FE engine model were extracted.

4.1 In-Cylinder Pressure and Temperature

The evolution of the trapped gas temperature and pressure is fundamentally tied to the piston kinematics and the Wiebe heat-release profile. As shown in Figure 6 and Figure 7, the compression stroke steadily increases both metrics until the spark timing at 350°.

Following ignition, the rapid heat release from combustion drives the temperature to a peak value shortly after Top Dead Center (TDC). This thermal spike, combined with the momentarily small cylinder volume, produces the peak firing pressure. As the piston descends during the expansion stroke, the continuous volume increase and convective heat losses to the cylinder walls cause both pressure and temperature to decay until the Exhaust Valve Opens (EVO) at 500°.

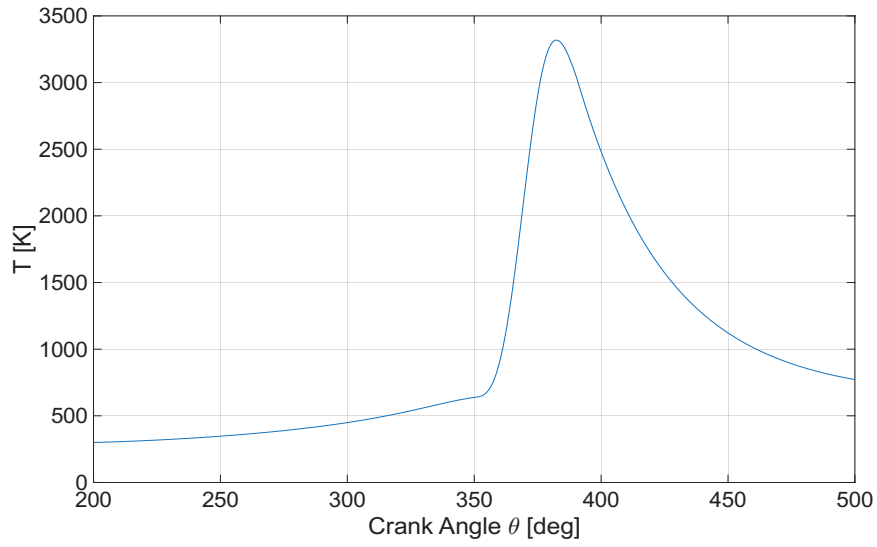


Figure 6: In-cylinder temperature history

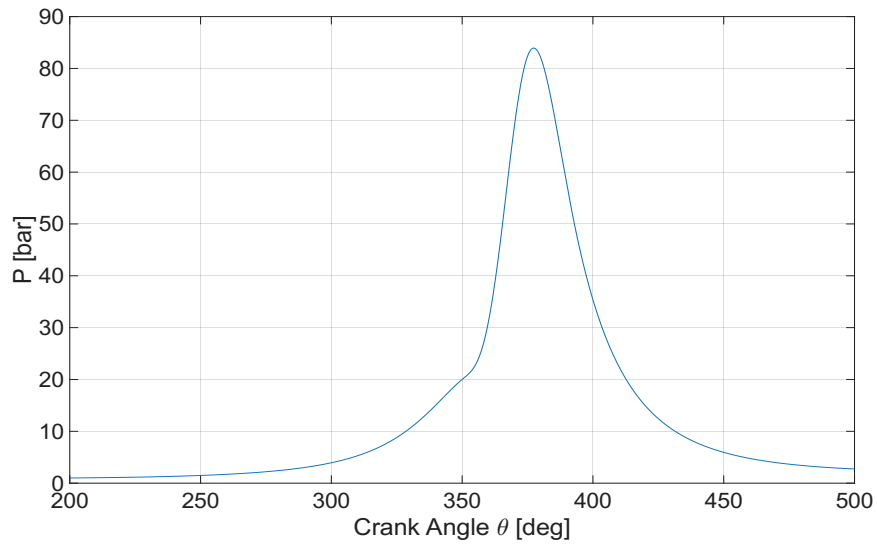


Figure 7: In-cylinder pressure history

4.2 Pressure-Volume (P-V) Diagram

While the crank-angle domain provides insight into transient behaviors, the thermodynamic performance of the engine is best evaluated using a Pressure-Volume (P-V) diagram. Figure 8 plots the cylinder pressure on a logarithmic scale against the normalized cylinder volume (V/V_c) on a linear scale.

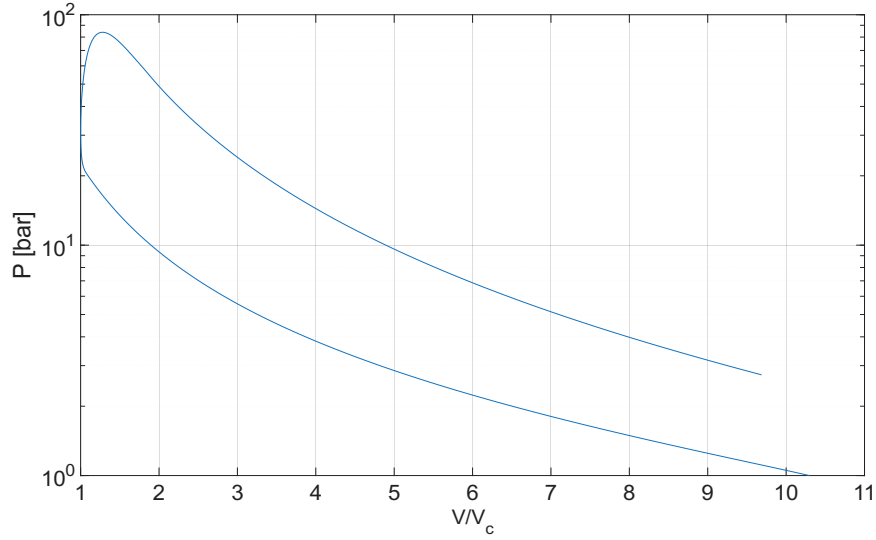


Figure 8: Normalized P-V Diagram (semi-logarithmic scale)

The area enclosed within this loop represents the gross indicated work (W_i) produced by the engine per cycle. On this semi-logarithmic projection, the compression and expansion strokes appear as curved segments rather than straight lines, since a linear volume axis does not linearize the polytropic relation $PV^n = \text{const}$; a fully log-log projection would be required to reveal that underlying linearity. The sharp near-vertical rise visible around TDC reflects the rapid pressure increase driven by the finite-duration Wiebe combustion model, occurring over a comparatively small change in cylinder volume.

4.3 Indicated and Brake Performance

By integrating the area inside the P-V loop, the gross indicated work (W_i) was calculated, yielding an Indicated Mean Effective Pressure (IMEP) and indicated power for the cylinder. However, to evaluate the usable output of the engine, mechanical friction must be subtracted.

Using the empirical friction model based on mean piston speed, the Friction Mean Effective Pressure (FMEP) was calculated.

The resulting Brake Mean Effective Pressure (BMEP) and Brake Power (P_b) at 3000 RPM represent the true work delivered to the crankshaft. The complete set of simulated performance metrics for the baseline operating point is summarized in Table 2.

Table 2: Simulated Engine Performance Metrics at 3000 RPM per cylinder

Parameter	Value	Unit
Indicated Work (W_i)	468.9	J
Indicated Mean Effective Pressure (IMEP)	12.53	bar
Thermal Efficiency (η_{th})	33.3	%
Friction Mean Effective Pressure (FMEP)	1.80	bar
Brake Mean Effective Pressure (BMEP)	10.73	bar
Brake Torque (T_b)	32	Nm
Brake Power (P_b)	10	kW

To understand the engine's performance across its operating range, the engine speed was swept from 1000 RPM to 6000 RPM. The resulting brake power curve is presented in Figure 9. As expected, the power output scales near-linearly with engine speed in the mid-range.

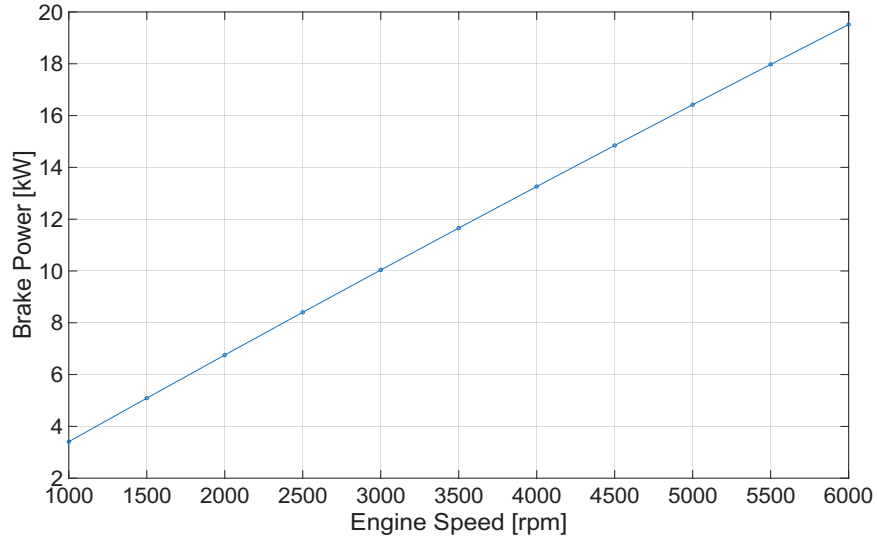


Figure 9: Brake power vs engine speed per cylinder (constant IMEP assumption)

5 Conclusion

This project successfully implemented a zero-dimensional (0D) single-zone thermodynamic model of a Toyota 1NZ-FE 1.5 L SI engine using MATLAB. By coupling the fundamental slider-crank kinematics with a Wiebe heat-release sub-model and Woschni heat transfer correlations, the simulation effectively captured the transient in-cylinder pressure and temperature histories over the closed-valve interval of the engine cycle, from IVC (200°) to EVO (500°).

The explicit Euler integration of the First Law of Thermodynamics produced a physically accurate pressure-volume (P-V) relationship. From this thermodynamic loop, the gross indicated work and Indicated Mean Effective Pressure (IMEP) were extracted. The application of an empirical friction mean effective pressure (FMEP) model allowed for the translation of these indicated metrics into realistic brake performance data, demonstrating a predictable power curve that scales with engine speed until mechanical friction dominates at the upper limits.

While this uncalibrated baseline model provides robust physical insight into the engine's thermodynamic cycle, it relies on static boundary conditions. Future iterations would benefit from integration with a 1D gas-exchange model to capture volumetric efficiency variations caused by fluid inertia, as well as experimental dynamometer data to precisely calibrate the global Woschni heat transfer coefficient.

References

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- [2] Woschni, Gerd, "A Universally Applicable Formula for the Instantaneous Heat Transfer Coefficient in the Internal Combustion Engine," SAE 670931, 1967.
- [3] Toyota Motor Corporation, "1NZ-FE Engine Specifications," Technical Service Manual, 2019.